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Submitted to:

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Neutrino Masses or New Interactions

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June 6, 2001

Abstract

Recent proposals to study the mass of the "electron" neutrino at a sensititvity of 0.3 eV can be used to place limits on the right handed and scalar charged currents at a level which improves on the present experimental limits. Indeed the neglect of the possibility of such interactions can lead to the inference of an incorrect value for the mass, as we illustrate.

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1 Introduction

Understanding the properties of the neutrino is one of the foremost problems in modern particle physics, in both the experimental and theoretical arenas. Of prime importance is the determination of the neutrino mass eigenstates: indications from neutrino oscillation experiments are that at least one neutrino has a non-zero mass. The atmospheric neutrino data measured at the Super-Kamiokande experiment favors $\nu_\mu - \nu_\tau$ oscillation with a mass difference of $\delta m^2_{atm} \sim 3 \times 10^{-3}$ [1]. The simplest interpretation of this is that one neutrino has a mass $m_3 \geq 0.05$ eV and as such is the first hint of physics beyond the Standard Model. Neutrino oscillation experiments indicate the finite value of the neutrino mass and the reveal how the mass eigenstates are mixed, however they cannot determine the absolute value of the neutrino masses, a question of equal significance.

The neutrino mass scale is accessible through the kinematics of weak interactions. A program of Tritium beta decay spectral measurements has been pursued to this end for a number of years [2]. Originally the focus of this program was to determine the Lorentz structure of Fermi's theory of weak interactions, that is Vector and Axial-vector (V-A) or some other combination of Scalar and Tensor currents[3]. More recently the focus has been solely on determining the mass of the electron anti-neutrino through precision measurements of the end-point of the electron energy spectrum [4] [6]. The current upper bound on the neutrino mass set by these experiments is $m_{\nu_e} \leq 2.5$eV while future experiments plan to achieve a sub-eV sensitivity of 0.3 eV, namely the KATRIN experiment to be developed at Mainz [5] [6]. The interpretation of the upper bound to the neutrino mass must be made with some care in light of the results from oscillation experiments, since as previously noted, these experiments are sensitive to the average mass of the mass eigenstates of the electron neutrino state [7] [8].

Many extensions to the minimal Standard Model, as required by the positive results of the oscillation experiments, introduce interactions with Lorentz structures other than V-A which have a coupling on a weaker scale than the Fermi coupling. This possibility must be included in the analysis of precision experiments such as the next generation Tritium decay experiments; the implications of not doing so have been examined previously in refs. [9] and [10] in the context of the negative mass squared anomaly of the electron anti-neutrino. In this note we examine the consequences for the measured value of the electron neutrino mass and the interpretation of this parameter in the presence of non-standard currents. IN particular we show that, in the presence of neutrino mixing, there may be interference effects which completely distort conclusions which may be drawn from the experiments concerning the nature of physics beyond the Standard Model.

2 Formalism

In this section we briefly outline a formalism used to describe nuclear beta decay in the presence of interactions other than the standard model V-A, which is described in greater depth in Ref. [9]. Low energy physics is best described by a current-current type of interaction (also called four fermion contact interaction). New physics at higher mass scales will manifest itself as additional currents possibly with different Lorentz symmetries from the dominant $V-A$ structure. These structures are S, P, T, V and A, *ie* Scalar, Pseudo-scalar, Tensor, Vector and Axial Vector currents respectively, and can be recast into their right and left handed components S_R, S_L, T, R and L with $L=(V-A)$, $R=(V+A)$, $S_R=(S+P)$ and $S_L=(S-P)$.

The most general effective interaction Hamiltonian for low energy, semi-leptonic decays is given by,

$$H_I = \sum_{\alpha,\beta=S_L,S_R,R,L,T} G^{\alpha\beta} \sum_f \left(J^{\dagger}_{h\alpha}.J_{f\beta} + h.c. \right) \tag{1}$$

where $f = e,\ \mu,\ \tau$, labels the weak eigenstate and in the Tritium case $J_{f\lambda=e\lambda} = \bar{\psi}_e \Gamma_\lambda \psi_{\nu_e}$ represents the leptonic current while $J_{h\alpha}$ represents the hadronic current. The operators Γ_λ are linear combinations of the five bilinear covariants,

$$\begin{aligned}
\Gamma_{S_L} &= (1-\gamma_5) \\
\Gamma_{S_R} &= (1+\gamma_5) \\
\Gamma_R &= \gamma^\mu(1+\gamma_5) \\
\Gamma_L &= \gamma^\mu(1-\gamma_5) \\
\Gamma_T &= [\gamma^\mu, \gamma^\nu]/2 \ .
\end{aligned} \tag{2}$$

In the Standard Model the only couplings present are $\beta = L$ and $\alpha = L, R$. These are expressed in more familiar notation as

$$G^{RL} + G^{LL} = V_{ud} \frac{\pi \alpha_W}{\sqrt{2} M_W^2} \ , \tag{3}$$

where V_{ud} is the element of the CKM matrix appropriat to nuclear beta decay, $\alpha_W = \frac{g_W^2}{4\pi}$ is the fine structure constant for weak interaction of the S.M. related to the coupling constant g_W, and M_W is the mass of the $W^{\pm}$.

The requirement that the Hamiltonian be Lorentz invariant causes most off diagonal terms in the sum to vanish, with the exception being (S_R, S_L)

and (R, L). Note also that in the case of β^--decay the subscripts of the operators S_R and S_L denote the chirality of the neutrino field and do not correspond to the chirality of electron field.

The indications from neutrino oscillations are that the weak eigenstate is not the same as the mass eigenstate. We define the weak eigenstates ν_f as linear combinations of the mass eigenstates,

$$\nu^f = \sum_i \cos\theta_i^f \ \nu_i \tag{4}$$

where the $cos\theta_i^f$ are the direction cosines in the coordinate system spanned by the mass eigenstates.

In general, should additional currents exist the boson mediating the current need not couple the eigenstates of the new interaction to the same linear combination of mass eigenstates; hence we define

$$\hat{\nu}^f = \sum_i \cos\theta_{iX}^f \ \nu_i \tag{5}$$

where the subscript 'X' corresponds to the type of interaction and takes the values S_R, S_L, R, or T. The angles 'θ_i^f'in the analysis of oscillation experiments can be taken to lie in the first quadrant without loss of generality, however relative to the new angles 'θ_{iX}^f' this is no longer so.

The new lorentz structures will produce interference terms in the beta decay spectra proportional to the coupling constants $G^{\alpha\beta}$. It is these interference effects which are used to produce the best limits on the strength of the new couplings [11]. For a full description of the structure of the interference terms see ref [9], note also that the interference effects due to the tensor (T) interaction with the left-chiral interaction are the same as that of the scalar interactions and will not be discussed further.

The strength of interference effects are usually evaluated under the assumption of that the weak eigenstate is the same as the mass eigenstate, and quoted relative to the strength of the S.M. coupling as,

$$\rho_X = \frac{\hat{g}^2 M_W^2}{g^2 M_X^2} \tag{6}$$

where $\hat{g}$ and M_X are the coupling constant and mass of the non-S.M. boson being exchanged and are related to the couplings of Eq. 1 by an analogous relationship to Eq. 3. The subscript X is the type of interaction: S_R, S_L or R. If the assumption that the weak and mass eigenstates are the same is relaxed, the strength of new interactions must be evaluated for each mass eigenstate on an individual basis. The constraints should now be taken to mean $\rho_{\hat{X}} = \rho_X \cos\theta_k \cos\theta_X$.

Recent limits are given as [12]

$$\begin{aligned} \rho_R &\leq 0.07 \\ \rho_{S_R} &\leq 0.1 \\ \rho_{S_L} &\leq 0.01 \end{aligned}$$

where the equivalence of the mass and weak eigenstates has been assumed. The constraints on the left-chiral scalar interaction are stronger than the others due to the fact that this current would produce a charged lepton of the wrong chirality. Because of this strong constraint we do not consider this interaction further.

3 Spectra

In this section we show how new Lorentz structures will be detected in the various experimental spectra.

3.1 Differential spectra and Kurie plots.

The differential electron energy spectrum with scalar and right-handed currents involves a sum over atomic final states, i, and is given by,

$$\begin{aligned} \left(\frac{dN}{dE_\beta}(\mathcal{E}_0^i)\right) &= KF(E_\beta)q_\beta(\mathcal{E}_0^i - E_\beta)\sum_k \Theta(\mathcal{E}_0^i - E_\beta - m_k) \\ &\times E_\beta(\mathcal{E}_0^i - E_\beta)\sqrt{1 - \frac{m_k^2}{(\mathcal{E}_0^i - E_\beta)^2}} \\ &\times \left([\cos^2\theta_k + \cos^2\theta_{kR}\rho_R^2 + \left(\frac{G_V^2}{G_V^2 + 3G_A^2}\right)\cos^2\theta_{kS_R}\rho_{S_R}^2]\right. \\ &- 2m_e m_k[\cos\theta_k\cos\theta_{kR}\rho_R] \\ &- \left. m_k E_\beta\left(\frac{G_V^2}{G_V^2 + 3G_A^2}\right)[\cos\theta_k\cos\theta_{kS_R}\rho_{S_R}]\right) \end{aligned} \tag{7}$$

The neutrino energy is defined by the difference between the end point energy ($\mathcal{E}_0^i$) and the electron energy ($E_\beta = \sqrt{q_\beta^2 + m_\beta^2}$) as $E_\nu = \mathcal{E}_0^i - E_\beta$. The Fermi function is given by $F(E_\beta)$ and m_k is the mass of the k^{th} neutrino mass eigenstate. Equation 7 can be bought into a more convenient form by noticing that near the end point the dependence on E_β is very weak and that the product $F(E_\beta)q_\beta$ is nearly constant. Thus a new constant can be defined;

$$K' = KF(E_\beta)q_\beta \ . \tag{8}$$

Further define,

$$\epsilon_{kR} = \rho_R \frac{\cos\theta_{kR}}{\cos\theta_k} \ , \tag{9}$$

and in the scalar case,

$$\epsilon_{kS_R} = \rho_{S_R} \frac{\cos\theta_{kS_R}}{\cos\theta_k} \ , \tag{10}$$

now define,

$$\epsilon_k = \epsilon_{kR}^2 + \epsilon_{kS_R}^2 (\frac{G_V^2}{G_V^2 + 3G_A^2}\Big) \tag{11}$$

and finally

$$\phi_k = -2\frac{m_k}{(1+\epsilon_k)}\Big[\frac{m_e}{< E_\beta >}\epsilon_{kR} + \epsilon_{kS_R}(\frac{G_V^2}{G_V^2+3G_A^2}\Big)\Big] \ . \tag{12}$$

The ratio $\frac{m_e}{E_\beta}$ varies by less than one part in a thousand over the whole spectrum so can safely be included at its average value in Equation 12. The differential spectrum is now

$$\begin{aligned}\left(\frac{dN}{dE_\beta}(\mathcal{E}_0^i)\right) &= K' \sum_k (1+\epsilon_k)\cos^2\theta_k(\mathcal{E}_0^i - E_\beta)^2(1+\epsilon_k)\left[1 + \frac{\phi_k}{(\mathcal{E}_0^i - E_\beta)}\right] \\ &\times \sqrt{1 - \frac{m_k^2}{(\mathcal{E}_0^i - E_\beta)^2}}\Theta(\mathcal{E}_0^i - E_\beta - m_k) \ . \end{aligned} \tag{13}$$

The effects of a finite neutrino mass on the differential spectrum is to cause a distortion near the end point, as the spectrum falls off as $E_\nu q_\nu$ rather than E_ν^2, and to shift the end point by m_ν ($\mathcal{E}_0 \rightarrow \mathcal{E}_0 - m_\nu$). Neutrino mixing has a similar result, producing several kinks in the spectrum at the point where the electron energy is such that decays to the k^{th} mass eigenstate are kinematically disallowed [13] [14].

3.2 Integral spectra

In the past many of the Tritium decay experiments performed were differential measurements, however more recently and in the future experiments measure an integral spectrum. That is they count the number of electrons above some cut off energy E_β^C,

$$N_i(E_\beta^C) = \int_{E_\beta^C}^{\infty} \frac{\mathrm{d}N_i}{\mathrm{d}E_\beta}\mathrm{d}E_\beta \ . \tag{14}$$

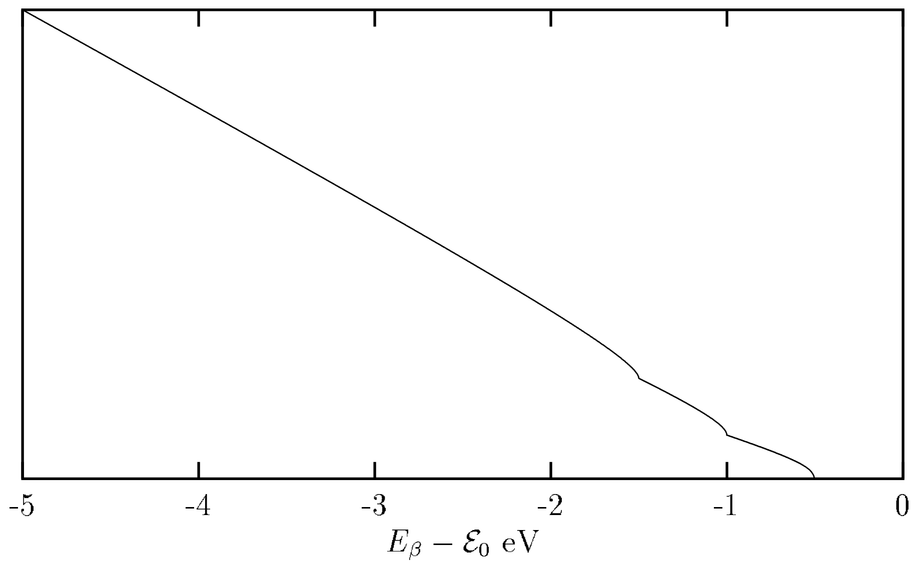

Figure 1: The last 5 eV of a Kurieplot for a fictitious mixing scheme. The three mass eignestates of $m_1 = 0.5$, $m_2 = 1.0$ and $m_3 = 1.5$ eV are maximally mixed.

In the Mainz experiment results for a number of different values of E_β^C are combined in a weighted average, the data is then fitted as a function of $E_l < E_\beta^C$, a lowest cutoff energy. The task of weighting the average takes into account the systematics of the spectrometer [4]. While we do not attempt to reproduce the details of this calculation we can gain insight from a crude model of this process, in which we integrate the integral spectrum from E_l to infinity,

$$\begin{aligned} G_i(E_l) &= \int_{E_l}^{\infty} N_i(E_\beta^C) dE_\beta^C \\ &= K' \sum_k (1+\epsilon_k)\cos^2\theta_k \left(\frac{1}{12}\left[E_{\nu_l}(E_{\nu_l}^2 - m_k^2)^{\frac{3}{2}} - \frac{3}{2} m_k^2 E_{\nu_l}(E_{\nu_l}^2 - m_k^2)^{\frac{1}{2}} \right.\right. \\ &+ \left. \frac{3}{2} m_k^4 \cosh^{-1}(E_{\nu_l}/m_k) \right] + \frac{\phi_k}{2}\left[\frac{1}{3}(E_{\nu_l}^2 - m_k^2)^{\frac{3}{2}} \right. \\ &- \left.\left. m_k^2 E_{\nu_l} \cosh^{-1}(E_{\nu_l}/m_k) - m_k^2 (E_{\nu_l}^2 - m_k^2)^{\frac{1}{2}} \right] \right) \end{aligned} \tag{15}$$

where in the last expression we have changed variables to $E_{\nu_l} = \mathcal{E}_0^i - E_l$, representing the neutrino energy in the i^{th} channel. In order to gain some understanding of the behavior of the equation we restrict our investigations to neutrino energies well below the end point, $E_{\nu_l} \gg m_k$, with this approximation the double integral becomes,

$$G_i(E_l) \approx \frac{K'}{12} \sum_k (1+\epsilon_k) E_{\nu_l}^4 \left[1 - 3\frac{m_k^2}{E_{\nu_l}^2} + 2\frac{\phi_k}{E_{\nu_l}} \right] . \tag{16}$$

The approximation is accurate to about one part in a thousand for cutoff energies $E_{\nu_l} > 35$ eV, and deteriorates rapidly for energies $E_{\nu_l} < 10$eV. To simplify the discussion we examine the case of a decay to one final sate of the atomic/molecular system by setting $i = 0$. Further we restrict the discussion to only one mass eigenstate $k = 1$. With these simplifications the function to be fitted to the data will be

$$G_0(E_l) \approx \frac{\mathcal{A}}{12} \left[E_{\nu_l}^4 - 3E_{\nu_l}^2 m_1^2 + 2E_{\nu_l}^3 \phi_1 + \mathcal{B} E_{\nu_l} \right] . \tag{17}$$

The coefficients of the mass and interference terms will not, due to the approximation made in Equation 16, be exactly 2 and -3, they will however be fixed and are not free to be fitted. The overall amplitude $\mathcal{A} = (1+\epsilon_1)\cos^2\theta_1$ will be determined from the data, as will the end point $\mathcal{E}_0^0$. The the background $\mathcal{B}$ can be independently determined from measurements far from the end point.

From a set of a measured values of $G_0(E_l)$ for different E_l, and for fixed $\mathcal{A}$, $\mathcal{B}$ and $\mathcal{E}_0^0$ the contours of equal likelihood will trace approximate ellipses in the (m_1^2, ϕ_1) plane. The major axes of the ellipses will have a positive slope as, at fixed E_l the doubly integrated spectrum is sensitive to $1.5m_1^2 - \phi_1 E_l$, rather than the orthogonal combination. As different values of the parameters $\mathcal{A}$, $\mathcal{B}$ and $\mathcal{E}_0^0$ are tried the center of the ellipse, as projected on to the (m_1^2, ϕ_1) plane moves over the plane. The curves of equal likelihood are thus going to be quite complex if the parameters $\mathcal{A}$, $\mathcal{B}$ and $\mathcal{E}_0^0$ are optimised at fixed m_1^2, ϕ_1, and the equal likelihood contours plotted in these remaining variables.

In fitteing the data it is important to be sure that all of the parameters, including $\mathcal{A}$, $\mathcal{B}$ and $\mathcal{E}_0^0$ are physically reasonable, and then to find a minimum χ^2 at a point on the (m_1^2, ϕ_1) plane consistent with the independent limits on ϵ_k. Discussions of the experimental results of future experiments should include a discussion of the values of all of the fitted parameters.

We now generalize to the case where all mass eigenstates participate in the decay. The KATRIN experiment should be sensitive to a mass of 0.3 eV, or about a 0.02% shift in $G_0(E_l)$ at an energy of $E_{\nu_l} \approx 40$ eV. For the various solutions to the solar and atmospheric neutrino problem this threshold may be reached depending on the mass scale [8]. The influence of additional interactions however, is uncertain, we investigate this by assuming the solutions

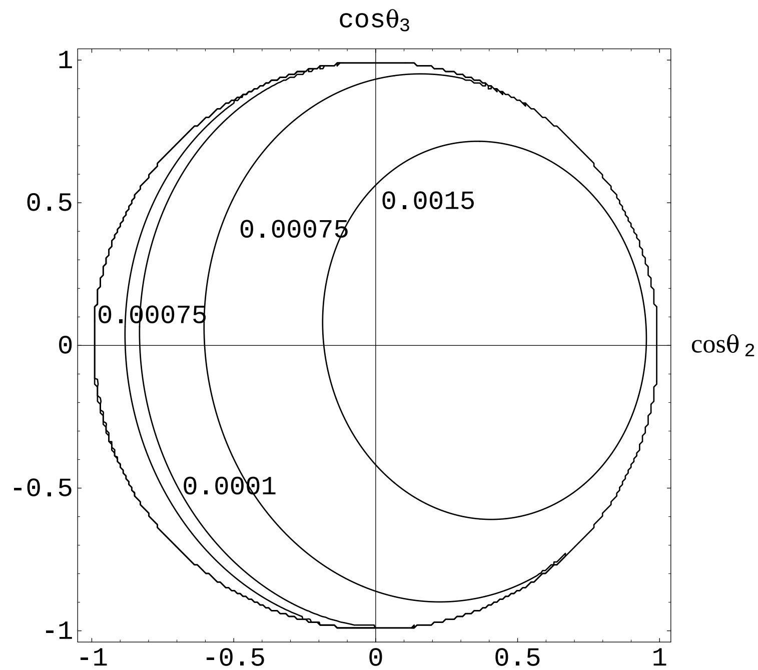

Figure 2: Constant value contours of $|\Delta G|$ for the right-chiral scalar interaction as compared to the spectrum produced assuming no additional interactions and the LMA solution of the solar neutrino data. In this plot the third mass eigenstate has a mass of $m_3 = 1$ eV. The numbers immediately to the right of each contour give the value of $|\Delta G|$.

to the oscillation data fix the values of $\cos\theta_k$ and also set the mass differences between the mass eigenstates. The value of $G_0(E_{\nu_l})$ will then be determined by the coupling strength and mixing of the additional interactions. To investigate the magnitude of this influence the absolute value of the relative difference between the spectra with and without (G_0^X and G_0' respectively) right-chiral scalar and right-handed vector couplings over the space of the non-standard direction cosines have been plotted. That is contours of;

$$|\Delta G| = \left|\frac{G_0^X - G_0'}{G_0^X}\right| \tag{18}$$

in the $(\cos\theta_{2X},\ \cos\theta_{3X})$ plane are shown in Figs 2 and 3. In this example the standard direction cosines are defined by Large Mixing Angle (LMA) solution to the solar neutrino problem, as are the mass diferences.

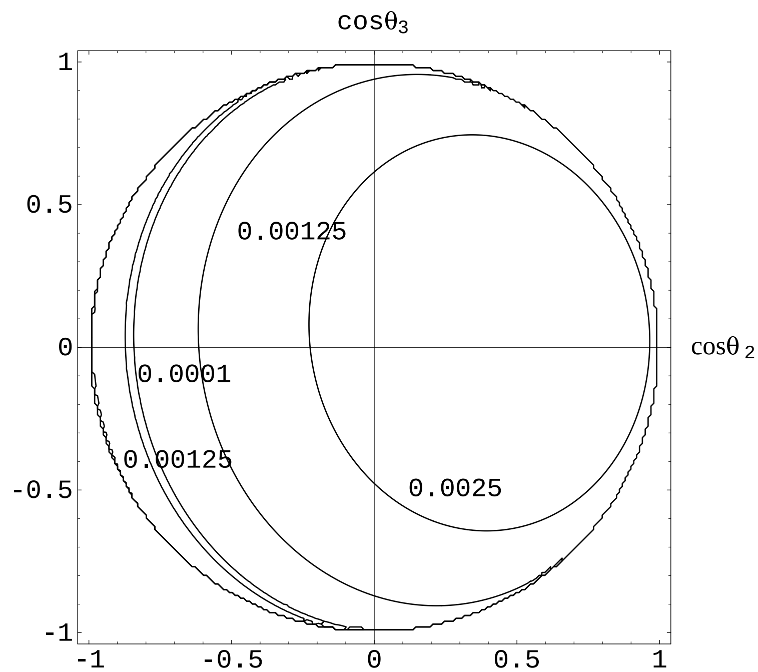

Figure 3: Constant value contours of $|\Delta G|$ for the right-handed vector interaction as compared to the spectrum produced assuming no additional interactions and the LMA solution of the solar neutrino data. In this plot the third mass eigenstate has a mass of $m_3 = 1$ eV. The numbers immediately to the right of each contour give the value of $|\Delta G|$.

The mass scale is chosen so as not to conflict with cosmological bounds [15], in this context it would be inappropriate to use bounds from the Tritium decay experiments. In producing these plots unitarity for both the standard and non-standard mixing cosines has been assumed. The amplitude for the spectra will be experimentally determined, and as such, will be independent of assumptions made about the additional interactions and is defined as $\mathcal{A} = K' \sum_k (1+\epsilon_k^2)\cos\theta_k^2$.

We note that for some values of $\cos\theta_{kX}$ there is no difference between the standard and non-standard spectra as a result of destructive interference between different mass eigenstates. This is an important point to consider in discussions about constraints on mass scales of new physics derived from the non-observation of interference effects in spectra or correlation parameters. The magnitude of the relative difference between spectra for some values $\cos\theta_{kX}$ is larger than the 0.02% detection threshold. An assumption of a particular standard mixing scheme in this situation would result in an incorrect fit for the value of m_{ν_e}. This reinforces the need for the possibility of weaker interactions in beta-decay to be taken into account when analysing future precision results.

4 Conclusion

In this note we have tried to highlight the important role that interactions other than the dominant $V-A$ interaction may play in precision measurements of the Tritium beta decay spectrum. If these are not correctly accounted for, fits to the neutrino mass will be missleading, and a door to new physics may be closed. The model for the integral spectra we have presented gives some insight into the likely effects of new interactions. Discussions of future experimental results at the expected level of sensitivity should include a full account of fits with the interference terms incorporated into the analysis.

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